Written Exam Economics Summer 2017

ADVANCED MACROECONOMETRICS

June 10, 10:00 – June 12, 10:00

This exam question consists of 10 pages in total

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Practical Information

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 1-5 with 19 questions to be answered. *Please answer all questions.*
- The exam paper should not exceed 20 pages. A maximum of 10 pages of supporting material (graphs, estimation output, etc.) can accompany the paper.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded to Digital Exam at the given time.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

1 Background

This project examination considers models for the business-cycle linkages between six countries in the period after 1980. We have quarterly observations for the period 1980:1-2016:4 on the following variables:

$U(A)_t$	:	The unemployment rate in country A, which is the largest member of the trade union NORTH.
$U(B)_t$	:	The unemployment rate in country B, which is a member of NORTH.
$U(C)_t$	:	The unemployment rate in country C, which is a member of NORTH.
$U(K)_t$	:	The unemployment rate in country K, which is the largest member of the trade union SOUTH.
$U(L)_t$	:	The unemployment rate in country L, which is a member of SOUTH.
$U(M)_t$	:	The unemployment rate in country M, which is a member of SOUTH.

All unemployment rates, $U(i)$, $i \in \{A, B, C, K, L, M\}$, are measured as percentages of the labour force, and we define the vector of variables,

$$x_t = (U(A)_t : U(B)_t : U(C)_t : U(K)_t : U(L)_t : U(M)_t)' . \tag{1.1}$$

Regarding the institutional setup, the countries in the trading union NORTH, i.e. A, B, and C, have experienced an increased integration over the considered period, and in the year 2002, they implemented a historic liberalization of the rules for international mobility of goods, capital, and labour. In addition, the economies where hit by a number of oil price shocks, the largest ones in 1993 and 2006, and these shocks have had implications on the labour markets of all countries, A, B, C, K, L, and M, to different degrees.

[1] Consider a vector of variables, $x_t \in \mathbb{R}^p$, and consider the set of equations

$$\Delta x_t = \alpha\beta' x_{t-1} + \mu + \epsilon_t, \quad t = 1, 2, ..., T, \tag{1.2}$$

where ϵ_t is independently and identically distributed, α and β are $p \times r$ matrices such that $\alpha'_\perp \beta_\perp$ has full rank $p - r$, μ is a $p \times 1$ vector, and x_0 is fixed.
Derive the solution for x_t as a function of x_0, the sequence $\{\epsilon_i\}_{i=1}^t$, and the parameters, $\{\mu, \alpha, \beta\}$.
Explain why it is important that $\alpha'_\perp \beta_\perp$ has full rank and what happens to your derivations if $\beta = \alpha_\perp$.

A macro-economist has argued that the unemployment rates of the six countries over the considered period have been driven by a single *global business-cycle factor*, and he has

suggested a simple representation of the form

$$\begin{pmatrix} U(A)_t \\ U(B)_t \\ U(C)_t \\ U(K)_t \\ U(L)_t \\ U(M)_t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \left(\mathsf{F}_t^{Global}\right) + S_t + A, \tag{1.3}$$

where $\mathsf{F}_t^{Global} = \sum_{i=1}^{t} e_{1i}$ is a simple random-walk representation of the very persistent global business-cycle factor with e_{1t} i.i.d., S_t is a stationary process that describes the transitory differences between the countries and A is a constant.

[2] Explain how the scenario in (1.3) is related to the equations in (1.2).
Explain what the implied cointegration rank, r, would be and how the cointegrating relations, i.e. the columns of β, would look.
Explain which restriction on μ you have to impose in (1.2) to get only a constant and no linear trends in the scenario in (1.3).

Other economists, however, have argued that the two trading areas, NORTH and SOUTH, are so separated, that they each have their own *local business-cycle factors*, and there has been some consensus on a model of the form,

$$\begin{pmatrix} U(A)_t \\ U(B)_t \\ U(C)_t \\ U(K)_t \\ U(L)_t \\ U(M)_t \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0.5 & 0 \\ 1 & 0.25 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0.5 \\ 1 & 0 & 0.25 \end{pmatrix} \begin{pmatrix} \mathsf{F}_t^{Global} \\ \mathsf{F}_t^{North} \\ \mathsf{F}_t^{South} \end{pmatrix} + S_t + A, \tag{1.4}$$

where $\mathsf{F}_t^{North} = \sum_{i=1}^{t} e_{2i}$, $\mathsf{F}_t^{South} = \sum_{i=1}^{t} e_{3i}$, and the magnitudes of the loading coefficients are suggested by the size of the countries and the magnitude of trading flows.

[3] Explain that the cointegration rank in this case is $r = 3$, and show that the coingration vectors could be chosen as

$$\beta = \begin{pmatrix} 1 & 0 & 1 \\ -3 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -3 & 2 \\ 0 & 2 & 0 \end{pmatrix}. \tag{1.5}$$

Now, modify the scenario in (1.4) such that the countries in the second trading area, SOUTH, are only affected by the global business cycle factor (and the system is driven by only two factors, F_t^{Global} and F_t^{North}). Show how the cointegration matrix, β, would change.

2 The Statistical Model

Now consider the vector in (1.1), such that $x_t \in \mathbb{R}^6$, and consider the vector autoregression

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \phi D_t + \epsilon_t, \quad t = 1, 2, ..., T, \tag{2.1}$$

where ϵ_t is assumed to be independently Gaussian distributed, $N(0, \Omega)$, the initial values, $x_{-k+1}, ..., x_{-1}, x_0$, are considered fixed for the statistical analysis and the vector D_t contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis.

[4] Perform a graphical analysis by considering the given time series and relevant linear combinations, and give a brief description of the economic development in the two trading areas.

[5] Set up and estimate a relevant empirical model for the data in (1.1). Carefully explain the steps you take and motivate the choices you make. In particular, you should motivate your choice of deterministic variables and discuss the potential presence of shifts in the equilibrium means.

[6] State the assumptions for the model, and test that the estimated model fulfills the assumptions.
In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

3 The Cointegration Rank

[7] Rewrite your preferred VAR model to error-correction form, and argue that for the model to have a unit root in the characteristic polynomial, the parameter

$$\Pi = \sum_{i=1}^{k} \Pi_i - I_p, \tag{3.1}$$

has to have reduced rank.

[8] Consider the likelihood ratio (trace) test statistic for the hypothesis

$$H(r) : \text{Rank}(\Pi) \leq r \quad \text{against} \quad H(p) : \text{Rank}(\Pi) \leq p. \tag{3.2}$$

Explain how to calculate the likelihood ratio statistic.
Explain how to implement a bootstrap version of the test for $H(r)$.
Now consider a particular bootstrap resampling scheme, where the bootstrap shocks in replication b, $b = 1, ..., B$, are constructed as

$$\epsilon_t^* = \hat{\epsilon}_t \cdot w_t, \quad t = 1, 2, ..., T, \tag{3.3}$$

where $\hat{\epsilon}_t$ is the estimated residual in the model $H(r)$ and w_t is distributed as $N(0, 1)$. Explain the intuition why this bootstrap is robust with respect to unmodelled ARCH effects.

[9] Determine the cointegration rank, r, in your preferred model using all the relevant information in the data.
Explain how your conclusion relates to the different scenarios presented above.

[10] For your choice of the cointegration rank, r, what would you expect on the $p \cdot k$ eigenvalues of the matrix

$$\begin{pmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_k & 0 \\ I_p & 0 & \cdots & 0 & 0 \\ 0 & I_p & & \cdots & \vdots \\ \vdots & & \ddots & 0 & 0 \\ 0 & \cdots & 0 & I_p & 0 \end{pmatrix}. \tag{3.4}$$

What would be your interpretation if the largest eigenvalue was 1.025?
What would be your interpretation if you consistently experienced that for all values of the cointegration rank, r, the largest unrestricted eigenvalue was very close to unit circle, say 0.99?

4 Hypotheses and Identification

[11] Impose your preferred cointegration rank and estimate the cointegrated VAR model. Test if the two unemployment rate differentials

$$U(A)_t - U(B)_t \quad \text{and} \quad U(A)_t - U(C)_t, \tag{4.1}$$

are stationary. If your model allows for restricted deterministic terms, try to include them in the relations and redo the tests.
Imagine that stationarity is rejected in both cases, such that $U(A)_t - U(B)_t$ and $U(A)_t - U(C)_t$ are non-stationary. Is this enough information to conclude that $U(B)_t - U(C)_t$ is also non-stationary?
Now imagine the opposite, such that $U(A)_t - U(B)_t$ and $U(A)_t - U(C)_t$ are stationary. Is this enough information to conclude that $U(B)_t - U(C)_t$ is also stationary?

[12] Test if any of the endogenous variables can be excluded from all cointegrating relationships and explain what an excludable variable implies for the Granger representation.

Also test if any restricted deterministic terms can be excluded.

[13] Test the hypothesis that one of the stochastic trends consists of accumulated shocks to the difference in unemployment rates of the two leading countries, A and K, i.e.

$$\alpha'_{\perp}\sum_{i=1}^{t}\epsilon_i = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & a_{2,2} & a_{3,2} & 0 & a_{5,2} & a_{6,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_{2,(p-r)} & a_{3,(p-r)} & 0 & a_{5,(p-r)} & a_{6,(p-r)} \end{pmatrix}\sum_{i=1}^{t}\epsilon_i, \tag{4.2}$$

where ϵ_t contains the shocks to the six equations, and $a_{i,j}$ denotes a parameter.

[14] Explain how to test the hypothesis that a shock to a certain variable, e.g. a shock to $U(A)_t$, has only transitory effects in the system, and test the hypothesis for all variables in x_t.

[15] Based on the theoretical candidates and your chosen cointegration rank, identify the cointegrating relationships in the empirical model. Explain the approach you use and the steps you take in the process.

Give an economic interpretation of the long-run structure and the equilibrium adjustment.

[16] Calculate the parameters of the Granger representation for your identified model and interpret the pushing forces in the model.

Do the results seem to resemble the theoretical scenarios?

5 Extensions

[17] (Parsimonious Reduced Form) Now fix the long-run relationships at their estimated values, e.g.

$$\mathsf{ecm}_t = \begin{pmatrix} \hat{\beta} \\ \hat{\beta}_D \end{pmatrix}' \begin{pmatrix} x_t \\ d_t \end{pmatrix}, \tag{5.1}$$

where d_t includes restricted deterministic terms.

Consider the error-correction model

$$\Delta x_t = \alpha \cdot \mathsf{ecm}_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1}\Delta x_{t-(k-1)} + \phi D_t + \epsilon_t, \tag{5.2}$$

where D_t contains all the unrestricted deterministic terms.

Estimate the parsimonious reduced form, i.e. the model in (5.2) with all insignificant coefficients restricted to zero.

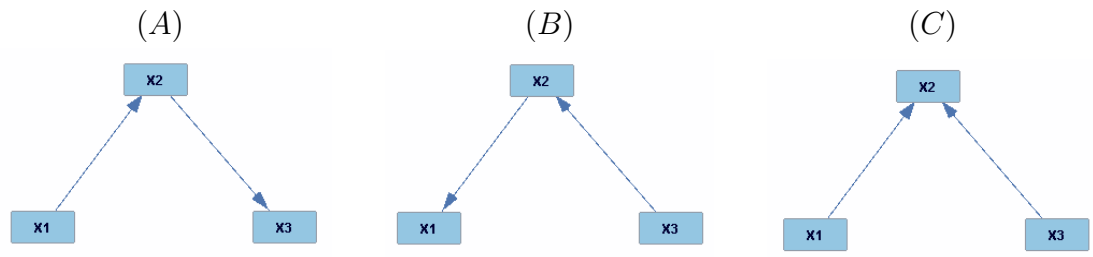
Comment on the residual correlations, and explain why residual correlation *may* reflect contemporaneous causal relationships between the variables, i.e. effects taking place within the same period.

[18] (Changing the Information Set) Imagine that you have performed the analysis with only five variables

$$x_t = (U(A)_t : U(B)_t : U(C)_t : U(K)_t : U(L)_t)'. \tag{5.3}$$

Based on your findings in the analysis above, explain what you would expect to find in terms of the cointegration rank, r, and in terms of the structure of the cointegration matrix, β.

[19] (Inference on Contemporaneous Causal Structures) Consider three variables, $X = (X_1 : X_2 : X_3)'$, and three graphical representations of their causal relationships



Explain how information on correlations and conditional correlations can be used to distinguish the three cases.

Next consider a set of six variables

$$X = (X_1 : X_2 : X_3 : X_4 : X_5 : X_6)'.$$

We want to infer the contemporaneous causal structure between the variables. The large table in the appendix reports correlations and conditional correlations between the six variables as well as $p-$values for the null hypothesis of a zero correlation (in brackets). Use this information to derive the class of observationally equivalent structures.

How many members does the class of observationally equivalent structures contain in this case?

Appendix

```
corr(X1,X2)             = -0.02 [0.44]
corr(X1,X3)             = -0.03 [0.42]
corr(X1,X4)             =  0.59 [0.00]
corr(X1,X4|X2)          =  0.77 [0.00]
corr(X1,X4|X2,X3)       =  0.91 [0.00]
corr(X1,X4|X2,X3,X5)    =  0.91 [0.00]
corr(X1,X4|X2,X3,X5,X6) =  0.65 [0.00]
corr(X1,X4|X2,X3,X6)    =  0.81 [0.00]
corr(X1,X4|X2,X5)       =  0.77 [0.00]
corr(X1,X4|X2,X5,X6)    =  0.42 [0.00]
corr(X1,X4|X2,X6)       =  0.59 [0.00]
corr(X1,X4|X3)          =  0.65 [0.00]
corr(X1,X4|X3,X5)       =  0.66 [0.00]
corr(X1,X4|X3,X5,X6)    =  0.25 [0.00]
corr(X1,X4|X3,X6)       =  0.43 [0.00]
corr(X1,X4|X5)          =  0.59 [0.00]
corr(X1,X4|X5,X6)       =  0.21 [0.00]
corr(X1,X4|X6)          =  0.36 [0.00]
corr(X1,X5)             =  0.01 [0.69]
corr(X1,X6)             = -0.51 [0.00]
corr(X1,X6|X2)          = -0.62 [0.00]
corr(X1,X6|X2,X3)       = -0.69 [0.00]
corr(X1,X6|X2,X3,X4)    =  0.02 [0.45]

corr(X2,X3)             = -0.01 [0.86]
corr(X2,X4)             =  0.62 [0.00]
corr(X2,X4|X1)          =  0.78 [0.00]
corr(X2,X4|X1,X3)       =  0.91 [0.00]
corr(X2,X4|X1,X3,X5)    =  0.91 [0.00]
corr(X2,X4|X1,X3,X5,X6) =  0.66 [0.00]
corr(X2,X4|X1,X3,X6)    =  0.80 [0.00]
corr(X2,X4|X1,X5)       =  0.78 [0.00]
corr(X2,X4|X1,X5,X6)    =  0.43 [0.00]
corr(X2,X4|X1,X6)       =  0.58 [0.00]
corr(X2,X4|X3)          =  0.67 [0.00]
corr(X2,X4|X3,X5)       =  0.67 [0.00]
corr(X2,X4|X3,X5,X6)    =  0.28 [0.00]
corr(X2,X4|X3,X6)       =  0.41 [0.00]
corr(X2,X4|X5)          =  0.62 [0.00]
corr(X2,X4|X5,X6)       =  0.24 [0.00]
corr(X2,X4|X6)          =  0.34 [0.00]
corr(X2,X5)             = -0.05 [0.15]
corr(X2,X6)             = -0.54 [0.00]
corr(X2,X6|X1)          = -0.65 [0.00]
corr(X2,X6|X1,X3)       = -0.72 [0.00]
corr(X2,X6|X1,X3,X4)    =  0.01 [0.81]
corr(X3,X4)             =  0.39 [0.00]
corr(X3,X4|X1)          =  0.50 [0.00]
corr(X3,X4|X1,X2)       =  0.81 [0.00]
corr(X3,X4|X1,X2,X5)    =  0.81 [0.00]
corr(X3,X4|X1,X2,X5,X6) =  0.57 [0.00]
corr(X3,X4|X1,X2,X6)    =  0.72 [0.00]
corr(X3,X4|X1,X5)       =  0.50 [0.00]
corr(X3,X4|X1,X5,X6)    =  0.16 [0.00]
corr(X3,X4|X1,X6)       =  0.30 [0.00]
corr(X3,X4|X2)          =  0.50 [0.00]
corr(X3,X4|X2,X5)       =  0.50 [0.00]
corr(X3,X4|X2,X5,X6)    =  0.17 [0.00]
corr(X3,X4|X2,X6)       =  0.29 [0.00]
corr(X3,X4|X5)          =  0.39 [0.00]
corr(X3,X4|X5,X6)       =  0.08 [0.01]
corr(X3,X4|X6)          =  0.18 [0.00]
corr(X3,X5)             = -0.01 [0.68]
corr(X3,X6)             = -0.35 [0.00]
corr(X3,X6|X1)          = -0.42 [0.00]
corr(X3,X6|X1,X2)       = -0.56 [0.00]
corr(X3,X6|X1,X2,X4)    = -0.00 [0.97]

corr(X4,X5)             = -0.03 [0.31]
corr(X4,X6)             = -0.88 [0.00]
corr(X4,X6|X1)          = -0.83 [0.00]
corr(X4,X6|X1,X2)       = -0.68 [0.00]
corr(X4,X6|X1,X2,X3)    = -0.47 [0.00]
corr(X4,X6|X1,X2,X3,X5) = -0.70 [0.00]
corr(X4,X6|X1,X2,X5)    = -0.86 [0.00]
corr(X4,X6|X1,X3)       = -0.79 [0.00]
corr(X4,X6|X1,X3,X5)    = -0.92 [0.00]
corr(X4,X6|X1,X5)       = -0.93 [0.00]
corr(X4,X6|X2)          = -0.82 [0.00]
corr(X4,X6|X2,X3)       = -0.77 [0.00]
corr(X4,X6|X2,X3,X5)    = -0.92 [0.00]
corr(X4,X6|X2,X5)       = -0.93 [0.00]
corr(X4,X6|X3)          = -0.86 [0.00]
corr(X4,X6|X3,X5)       = -0.95 [0.00]
corr(X4,X6|X5)          = -0.96 [0.00]
corr(X5,X6)             =  0.43 [0.00]
corr(X5,X6|X1)          =  0.51 [0.00]
corr(X5,X6|X1,X2)       =  0.63 [0.00]
corr(X5,X6|X1,X2,X3)    =  0.75 [0.00]
corr(X5,X6|X1,X2,X3,X4) =  0.84 [0.00]
corr(X5,X6|X1,X2,X4)    =  0.84 [0.00]
corr(X5,X6|X1,X3)       =  0.56 [0.00]
corr(X5,X6|X1,X3,X4)    =  0.84 [0.00]
corr(X5,X6|X1,X4)       =  0.84 [0.00]
corr(X5,X6|X2)          =  0.49 [0.00]
corr(X5,X6|X2,X3)       =  0.53 [0.00]
corr(X5,X6|X2,X3,X4)    =  0.84 [0.00]
corr(X5,X6|X2,X4)       =  0.84 [0.00]
corr(X5,X6|X3)          =  0.46 [0.00]
corr(X5,X6|X3,X4)       =  0.84 [0.00]
corr(X5,X6|X4)          =  0.84 [0.00]

corr(X1,X2|X4)          = -0.62 [0.00]
corr(X1,X3|X4)          = -0.34 [0.00]
corr(X1,X6|X4)          =  0.04 [0.25]
corr(X2,X3|X4)          = -0.34 [0.00]
corr(X2,X6|X4)          = -0.01 [0.69]
corr(X3,X6|X4)          = -0.02 [0.48]
corr(X4,X5|X6)          =  0.80 [0.00]
```